Units. — In the following formulae all lengths are expressed in centimeters. The inductance calculated will be in micro-henries = 10^{-6} henry.

Long straight round wire. — If l is the length; d, the diameter of cross section; μ the permeability of the material, — the inductance at zero or low frequency is,

$$L = 0.002\, l \left[2.303\, log\, \frac{4l}{d} - 1 + \frac{\mu}{4}\right]$$

For all except iron wire $\mu = 1$ and the last term becomes 0.25. For wires whose length is less than about 1000 times the diameter the term $+ d/(2l)$ should be added inside the brackets. For any frequency:

$$L = 0.002\, l \left[2.303\, log\, \frac{4l}{d} - 1 + \mu\delta\right]$$

where δ is a quantity given in Table 2 below as a function of x. x is to be computed from the relation

$$x = 0.1405\, d \sqrt{\frac{\mu f}{\rho}}$$

where d and μ are as above; f, the frequency and ρ the resistivity of the material of the wire expressed in microhm-centimeters. (See Properties of Metallic Conductors.) For copper at 20° C.

$$x = 0.1071\, d\sqrt{f}$$

For wires other than iron, whose length is 100,000 times the diameter the inductance at infinite frequency is about 2% less than at zero frequency.

TABLE 2

Values of δ for computing inductance at any frequency.

x	δ	x	δ
0	0.250	12	0.059
0.5	.250	14	.050
1.0	.249	16	.044
1.5	.247	18	.039
2.0	.240	20	.035
2.5	.228	25	.028
3.0	.211	30	.024
3.5	.191	40	.0175
4.0	.1715	50	.014
4.5	. 154	60	.012
5.0	.139	70	.010
6.0	.116	80	.009
7.0	. 100	90	.008
8.0	.088	100	.007
9.0	.078	co	.000
10.0	.070		

Two parallel round wires, return circuit.— If l is the length of each wire; d, the diameter; D, the distance between centers of wires; μ the permeability,— the inductance for any frequency is

$$L = 0.004\, l \left[2.303\, log\, \frac{2D}{d} - \frac{D}{l} + \mu\delta\right]$$

where δ is a quantity to be obtained from the table above as a function of x which is to be computed as explained for the previous formula.

For copper and at low frequency the term δ becomes 0.25.

Square of round wire.— If a is the length of the side of the square; d, the diameter of the wire; μ the permeability, the inductance for any frequency is,

$$L = 0.008\, a \left[2.303\, log\, \frac{2a}{d} - \frac{d}{2a} - 0.774 + \mu\delta\right]$$

where δ is obtained as above. For low frequency and for wires other than iron δ becomes 0.25; for infinite frequency the value is zero.

Grounded horizontal wire, the Earth acting as return circuit. If l is the length of wire; h, the height above the ground; d, the diameter of the wire; μ the permeability and δ the frequency constant (see table 2), the inductance,—where d is small compared with l,—is given as follows:

For $\frac{2h}{l} \leq 1 \qquad L = 0.002\, l \left[2.3026\, log\, \frac{4h}{d} - P + \mu\delta\right]$

For $\frac{l}{2h} \leq 1 \qquad L = 0.002\, l \left[2.3026\, log\, \frac{4l}{d} - Q + \mu\delta\right]$

P and Q may be found in the following table.

TABLE 3

$\frac{2h}{l}$	P	$\frac{l}{2h}$	Q		$\frac{2h}{l}$	P	$\frac{l}{2h}$	Q
0	0	0	1	0000	0.6	0.5136	0.6	1.2918
0.1	0.0975	0.1	1	0499	.7	.5840	.7	1.3373
.2	.1900	.2	1	0997	.8	.6507	.8	1.3819
.3	.2778	.3	1	1489	.9	.7139	.9	1.4251
.4	.3608	.4	1	1975	1.0	.7740	1.0	1.4672
5	.4393	.5	1	2452				

The mutual inductance of the case above may be expressed, For $\frac{2h}{l} \leq 1$ $L = 0.002\, l \left[2.3026\, log\, \frac{2h}{D} - P + \frac{D}{l}\right]$

For $\frac{l}{2h} \leq 1$ $L = 0.002\, l \left[2.3026\, log\, \frac{4l}{d} - Q + \frac{D}{l}\right]$

The values of P and Q are found in the table above.

Grounded wires in parallel. — Compute by the above formulae the inductance L_l per unit length of a single wire and the mutual inductance M_l per unit length of two adjacent wires, using the actual length in determining the ratios $2h/l$, $2l/d$ etc. Then the inductance of n parallel wires will be,

$$L = l \left[\frac{L_1 + (n-1)\, M_1}{n} - 0.001\, k\right]$$

where k is a function of n found in Table 1 under capacity formulae.

Circular ring of round wire. — If a is the mean radius of the ring; d, the diameter of the wire, the inductance at any frequency is

$$L = 0.01257\, a \left[2.303\, log\, \frac{16a}{d} - 2 + \mu\delta\right]$$

where δ is determined from the table above.

Circular coil of circular cross section. — For a coil of n fine wires wound with mean radius of the turns a, the cross section of whose winding is a circle of diameter d, the inductance at low frequency, for wire other than iron, neglecting insulation space is,

$$L = 0.01257\, an^2 \left[2.303\, log\, \frac{16a}{d} - 1.75\right]$$

Torus with a single layer transverse winding, — a circular solenoid of circular cross section. If r is the distance from the center of the torus to the center of the transverse section; a, the radius of the turns of the winding; n, the number of turns, the inductance at low frequency is

$$L = 0.01257\, n^2 \left[r - \sqrt{r^2 - a^2}\right]$$

Solenoid, single layer. If n is the number of turns; a the radius of the coil; b, the length, the approximate inductance at any frequency is,

$$L = \frac{0.03948\, a^2 n^2}{b} K$$

where K is a function of 2a/b given in the table below.

TABLE 4

$\frac{2a}{b}$	K	$\frac{2a}{b}$	K	$\frac{2a}{b}$	K
0.00	1.0000	2.00	0.5255	7.00	0.2584
.05	.9791	2.10	.5137	7.20	.2537
.10	.9588	2.20	.5025	7.40	.2491
.15	.9391	2.30	.4918	7.60	.2448
.20	.9201	2.40	.4816	7.80	.2406
.25	.9016	2.50	.4719	8.00	.2366
.30	.8838	2.60	.4626	8.50	.2272
.35	.8665	2.70	.4537	9.00	.2185
.40	.8499	2.80	.4452	9.50	.2106
.45	.8337	2.90	.4370	10.00	.2033
.50	.8181	3.00	.4292		
.55	.8031	3.10	.4217	11.0	.1903
.60	.7885	3.20	.4145	12.0	.1790
.65	.7745	3.30	.4075	13.0	.1692
.70	.7609	3.40	. 4008	14.0	.1605
.75	.7478	3.50	.3944	15.0	.1527
.80	.7351	3.60	.3882	16.0	.1457
.85	.7228	3.70	.3822	17.0	.1394
.90	.7110	3.80	.3764	18.0	.1336
.95	.6995	3.90	.3708	19.0	.1284
1.00	.6884	4.00	.3654	20.0	.1236
1.05	.6777	4.10	.3602	22.0	.1151
1.10	.6673	4.20	.3551	24.0	.1078
1.15	.6573	4.30	.3502	26.0	.1015
1.20	.6475	4.40	.3455	28.0	.0959
1.25	.6381	4.50	.3409	30.0	.0910
1.30	.6290	4.60	.3364	35.0	.0808
1.35	.6201	4.70	.3321	40.0	.0728
1.40	.6115	4.80	.3279	45.0	.0664
1.45	.6031	4.90	.3238	50.0	.0611
1.50	.5950	5.00	.3198	60.0	.0528
1.55	.5871	5.20	.3122	70.0	.0467
1.60	.5795	5.40	.3050	80.0	.0419
1.65	.5721	5.60	.2981	90.0	.0381
1.70	.5649	5.80	.2916	100.0	.0350
1.75	.5579	6.00	.2854		
1.80	.5511	6.20	.2795		
1.85	.5444	6.40	.2685		
1.90	.5379	6.60	.2739		
1.95	.5316	6.80	.2633		

Long multiple layer solenoid. — The inductance is given approximately by,

$$L = L_1 - \frac{0.01257\, n^2 ac}{b}(0.693 + B_s)$$

where L_1 is the inductance calculated from the formula for a single layer solenoid, n being the number of turns of the winding; a, the radius of the coil measured from the axis to the center of the cross section of the winding; b, the length of the coil; c, the radial depth of the winding; B_s, a correction given in table below as a function of b/c.

TABLE 5

b/c	B_s	b/c	B_s
1	0.0000	16	0.3017
2	.1202	17	.3041
3	.1753	18	.3062
4	.2076	19	.3082
5	.2292	20	.3099
6	.2446	21	.3116
7	.2563	22	.3131
8	.2656	23	.3145
9	.2730	24	.3157
10	.2792	25	.3169
11	.2844	26	.3180
12	.2888	27	.3190
13	.2927	28	.3200
14	.2961	29	.3209
15	.2991	30	.3218

Square coil of rectangular cross section. — If a be the side of the square measured to the center of the rectangular section which has sides b and c and if n be the number of turns,

$$L = 0.008an^2\left[2.303\ log\frac{a}{b+c} + 0.2235\frac{b+c}{a} + 0.726\right]$$

If the cross section is a square $b = c$ and the expression becomes

$$L = 0.008an^2\left[2.303\ log\frac{a}{b} + 0.447\frac{b}{a} + 0.033\right]$$

Mutual Inductance

Two parallel wires. — If I be the length of each wire; D, the distance between, the inductance is

$$L = 0.002l\left[2.303\ log\frac{2l}{D} - 1 + \frac{D}{l}\right]$$

Coaxial solenoids, single layer coils, not concentric. If a is the radius of the smaller coil; A, the radius of the larger: n_1 and n_2 the number of turns on the smaller and larger coil respectively; $2l$ the length of the smaller coil; $2x$, the length of the larger; *D,* the distance between the centers of the coils measured along the common axis,

$$M = 0.009870\frac{a^2A^2n_1n_2}{2x\ 2l}[K_1k_1 + K_3k_3 + K_5k_5]$$

Where

$$K_1 = \frac{2}{A^2}\left(\frac{x_1}{r_1} - \frac{x_2}{r_2}\right) \qquad k1 = 2l$$

$$K_3 = \frac{1}{2}\left(\frac{x_1}{r_1^5} - \frac{x_2}{r_2^5}\right) \qquad k_3 = a^2l\left(3 + 4\frac{l^2}{a^2}\right)$$

$$K_5 = -\frac{A^2}{8}\left[\frac{x_1}{r_1^9}\left(3 - 4\frac{x_1^2}{A^2}\right) - \frac{x_2}{r_2^9}\left(3 - 4\frac{x_2^2}{A^2}\right)\right] \qquad k_5 = a^4l\left(\frac{5}{2} - 10\frac{l^2}{a^2} + 4\frac{l^4}{a^4}\right)$$

Where

$$x_1 = D - x \qquad r_1 = \sqrt{x_1^2 + A^2}$$

$$x_2 = D + x \qquad r_2 = \sqrt{x_2^2 + A^2}$$

The above is most accurate for short coils with relatively great distance between.

Coaxial, concentric solenoids, outer coil the longer. If a be the radius of the smaller coil; A, that of the larger; $2l$, the length of the inner coil; $2x$, the length of the outer; n_1 and n_2 the number of turns on the inner and outer coil respectively,

$$M = \frac{0.01974a^2n_1n_2}{g}\left[1 + \frac{A^2a^2}{8g^4}\left(3 - 4\frac{l^2}{a^2}\right)\right]$$

where

$$g = \sqrt{x^2 + A^2}$$

Coaxial, concentric solenoids, outer coil the shorter. Assuming the symbols as before except

$$g = \sqrt{l^2 + A^2}$$

$$M = \frac{0.01974a^2n_1n_2}{g}\left[1 + \frac{A^2a^2}{8g^4}\left(3 - 4\frac{x^2}{a^2}\right)\right]$$

HIGH FREQUENCY RESISTANCE

Cylindrical straight wires. —The ratio R/R_0 of the high frequency resistance to the resistance at low frequency may be found from the table below, by calculating first the value of x from the relation,

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$$x = \pi d \sqrt{\frac{2\mu f}{\rho}} \sqrt{\frac{1}{1000}}$$

where d is the diameter of the wire in centimeters; μ, the magnetic permeability; f, the frequency; ρ, the resistivity in microhm-centimeters.

For copper wire $x = 10\ da$ where a has a value given by $a = 0.01071 \sqrt{f}$. The value of a for various frequencies may be found in the second of the two tables below. The above method gives the high-frequency resistance of simple circuits of any shape where the length is great compared with the diameter of the wire and the different portions of the circuit are not close to each other.

TABLE 6

Ratio of High-Frequency Resistance to the Direct-Current Resistance.

x	R/R_0	x	R/R_0	x	R/R_0
0	1.0000	5.2	2.114	14.0	5.209
0.5	1.0003	5.4	2.184	14.5	5.386
.6	1.0007	5.6	2.254	15.0	5.562
.7	1.0012	5.8	2.324	16.0	5.915
.8	1.0021	6.0	2.394	17.0	6.268
.9	1.0034	6.2	2.463	18.0	6.621
1.0	1.005	6.4	2.533	19.0	6.974
1.1	1.008	6.6	2.603	20.0	7.328
1.2	1.011	6.8	2.673	21.0	7.681
1.3	1.015	7.0	2.743	22.0	8.034
1.4	1.020	7.2	2.813	23.0	8.387
1.5	1.026	7.4	2.884	24.0	8.741
1.6	1.033	7.6	2.954	25.0	9.094
1.7	1.042	7.8	3.024	26.0	9.447
1.8	1.052	8.0	3.094	28.0	10.15
1.9	1.064	8.2	3.165	30.0	10.86
2.0	1.078	8.4	3.235	32.0	11.57
2.2	1.111	8.6	3.306	34.0	12.27
2.4	1.152	8.8	3.376	36.0	12.98
2.6	1.201	9.0	3.446	38.0	13.69
2.8	1.256	9.2	3.517	40.0	14.40
3.0	1.318	9.4	3.587	42.0	15.10
3.2	1.385	9.6	3.658	44.0	15.81
3.4	1.456	9.8	3.728	46.0	16.52
3.6	1.529	10.0	3.799	48.0	17.22
3.8	1.603	10.5	3.975	50.0	17.93
4.0	1.678	11.0	4.151	60.0	21.47
4.2	1.752	11.5	4.327	70.0	25.00
4.4	1.826	12.0	4.504	80.0	28.54
4.6	1.899	12.5	4.680	90.0	32.07
4.8	1.971	13.0	4.856	100.0	35.61
5.0	2.043	13.5	5.033		

As an extension of the above table the following relation may be used: $R/Ro = x/2.828+0.25$. The equation is valid for values of x greater than 7 at which point the error is about 1% and decreasing with increasing values of x

TABLE 7

Values of a (= .01071 $\sqrt{f}$) for various frequencies.

f	a	WAVE-LENGTH METERS	f	a	WAVE-LENGTH METERS
100	0.1071		50,000	2.395	6,000
200	.1514		60,000	2.624	5,000
300	.1855		70,000	2.834	4,286
400	.2142		80,000	3.029	3,750
500	.2395		90,000	3.213	3,333
600	.2624		100,000	3.387	3,000
700	.2834		150,000	4.148	2,000
800	.3029		200,000	4.790	1,500
900	.3213		250,000	5.355	1,200
1,000	.3387		300,000	5.866	1,000
2,000	.4790		333,333	6.184	900
3,000	.5866		375,000	6.564	800
4,000	.6774		428,570	7.012	700
5,000	.7573		500,000	7.573	600
6,000	.8296		600,000	8.296	500
7,000	.8960		700,000	8.960	429
8,000	.9579		750,000	9.275	400
9,000	1.0160		800,000	9.579	375
10,000	1.071	30,000	900,000	10.16	333
15,000	1.312	20,000	1,000,000	10.71	300
20,000	1.514	15,000	1,500,000	13.12	200
30,000	1.855	10,000	3,000,000	18.55	100
40,000	2.142	7,500			